Deep inelastic scattering in AdS/QCD

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Outline

- **Brief review of IR cut-off in AdS $\Rightarrow$ Hadronic Physics**

- **Deep inelastic scattering (DIS) in the hard wall model (review)**

- **DIS in the soft wall model**

- **DIS with final states with higher conformal dimensions**

• **Another strong motivation for Phenomenological duals to QCD based on IR cut offs (AdS/QCD):**

Polchinski and Strassler (PRL 2002)

Scaling of high energy hadron-hadron scattering amplitudes with fixed angles ($s \longrightarrow \infty$ with s/t fixed) from gauge/string duality.

$AdS_5 \times S^5$ **space**

$$ds^2 = \frac{R^2}{z^2}(dz^2 + (d\vec{x})^2 - dt^2) + R^2 d\Omega_5^2 \,,$$

with $0 \le z \le z_{max} = 1/\Lambda$ infrared (IR) cut-off (now called hard wall)

Using this simple IR cut-off they found the scaling for the glueball amplitudes.

$$A(p) \sim \left(\frac{\Lambda}{p}\right)^{\Delta - 4}$$

(Δ = total scaling dimension of *in* and *out* states)

$\Rightarrow$ Hard scattering as observed for hadrons (as opposed to string amplitudes in flat space-time that show soft scattering in this regime)

This scaling was also reproduced, after, using a mapping between scalar states in the AdS slice and on the boundary that preserves canonical commutation relations: H. Boschi-Filho and N. B. , PLB 2003.

Motivated by Polchinski and Strassler result:

- **Simple estimate for glueball mass ratios**

H. Boschi-Filho and N.B. , hep-th/0209080 ; hep-th/0212207.

Scalar states in AdS slice with Dirichlet boundary condition dual to scalar glueballs $J^{PC} = 0^{++}$, 0^{++*}, 0^{++**} , ... with masses μ_i. The lightest mass is related to the size of the slice.

Considering an approximate gauge/string duality we found glueball masses related to zeros of Bessel functions

$$\frac{\mu_i}{\mu_1} = \frac{\chi_{2\,,\,i}}{\chi_{2\,,\,1}}$$

Results consistent with lattice data available.

- **In 2002, estimating hadronic masses by just placing some IR cut-off in AdS was like a heresy but shortly this kind of approach became popular.**

The previous results were just for scalar glueballs.

Masses for glueball states with different spins and comparison with Pomerons: H. Boschi-Filho, N. B. and H. L. Carrion, PRD 2006.

Regge trajectory for Pomerons

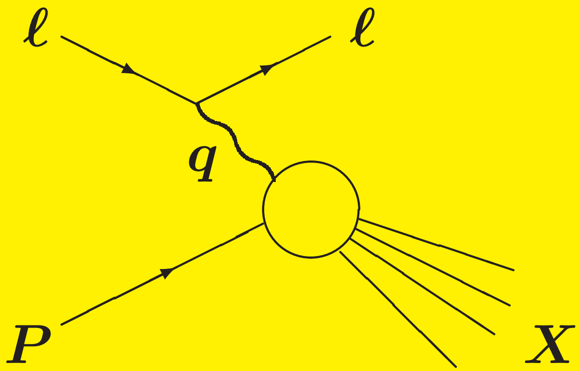
$$J \equiv \alpha_0 + \alpha' M^2 \approx 1.08 + 0.25\, M^2 \qquad (GeV)$$

(for baryons and mesons $J \approx 0.5 + (0.9) M^2$).

Pomerons may be related to glueballs. Recent lattice results are consistent with this interpretation (slope and intercept) H. B. Meyer and M. J. Teper, PLB 2005

For Neumann boundary conditions we found a linear fit compatible with the Pomeron trajectory:

$$\alpha' = (0.26 \pm 0.02) GeV^{-2} \qquad ; \qquad \alpha_0 = 0.80 \pm 0.40$$



Deep Inelastic Scattering (DIS)

Bjorken parameter: $x \equiv -q^2/2P \cdot q$.

DIS: $q^2 \to \infty$, with x fixed.

Hadronic tensor (for unpolarized scattering)

$$W^{\mu\nu} = i \int d^4y\, e^{iq\cdot y} \langle P, \mathcal{Q}|\, [J^\mu(y), J^\nu(0)]\, |P, \mathcal{Q}\rangle \,,$$

where $J^\mu(y)$ = hadron current and $\mathcal{Q}$ = electric charge.

Structure functions

$$W^{\mu\nu} = F_1(x,q^2)(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}) + \frac{2x}{q^2}F_2(x,q^2)(P^\mu + \frac{q^\mu}{2x})(P^\nu + \frac{q^\nu}{2x}),$$

DIS cross section is related to the Forward Compton scattering amplitude, determined by the tensor

$$T^{\mu\nu} = i\int d^4y e^{iq\cdot y}\langle P,\mathcal{Q}|\,\mathcal{T}(J^\mu(y)J^\nu(0))\,|P,\mathcal{Q}\rangle\,,$$

decomposed as

$$T^{\mu\nu} = \tilde{F}_1(x,q^2)(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}) + \frac{2x}{q^2}\tilde{F}_2(x,q^2)(P^\mu + \frac{q^\mu}{2x})(P^\nu + \frac{q^\nu}{2x}),$$

Optical theorem implies

$$F_{1,2}(x,q^2) \equiv 2\pi\,\mathrm{Im}\,\tilde{F}_{1,2}(x,q^2)\,.$$

Imaginary part of forward Compton scattering amplitude expressed in terms of sum over "intermediate" states X with mass M_X

$$\mathrm{Im}\, T^{\mu\nu} = 2\pi^2 \sum_X \delta(M_X^2 + (P+q)^2)\langle P, \mathcal{Q}|J^\nu(0)|P+q,\, X\rangle \langle P+q,\, X|J^\mu(0)|P, \mathcal{Q}\rangle \; .$$

• **Depends on the hadronic spectrum!**

DIS from gauge/string duality in the hard wall model
Polchinski and Strassler, JHEP 0305, 012 (2003)

Prescriptions for calculating $\mathrm{Im}\, T^{\mu\nu}$ from string theory for three different kinematical regimes of Bjorken parameter x:

- $x > 1/\sqrt{gN}$
- $\exp(-\sqrt{gN}) << x << (gN)^{-1/2}$
- $x < \exp(-\sqrt{gN})$.

In the first case the supergravity approximation is valid while in the other regimes massive string states contribute.

Prescription in the supergravity regime: relation between matrix elements of hadronic current and ten dimensional interaction action. For a scalar particle

$$\begin{aligned} \eta_\mu \langle P_X, X | \tilde{J}^\mu(q) | P, \mathcal{Q} \rangle &= (2\pi)^4 \, \delta^4(P_X - P - q) \, \eta_\mu \, \langle P + q, X | J^\mu(0) | P, \mathcal{Q} \rangle \\ &= i\mathcal{Q} \int d^{10}x \sqrt{-g} A^m (\Phi_i \partial_m \Phi_X^* - \Phi_X^* \partial_m \Phi_i) \ . \end{aligned}$$

($m = AdS_5$ coordinates; $\mu =$ boundary coordinates)

Result

$$F_1(x,q^2) = 0 \;\; ; \;\; F_2(x,q^2) = \pi C_0 \, \mathcal{Q}^2 \left(\frac{\Lambda^2}{q^2}\right)^{\Delta-1} x^{\Delta+1} (1-x)^{\Delta-2} ,$$

where Δ is the scaling dimension of the scalar states (initial = final) and $\mathcal{Q}$ is the charge.

For fermions they found

$$F_2(x,q^2) = 2F_1(x,q^2) = C \, \mathcal{Q}^2 \left(\frac{\Lambda^2}{q^2}\right)^{\tau-1} x^{\tau+1} (1-x)^{\tau-2} ,$$

where τ = twist.

• Soft Wall Model

A. Karch, E. Katz, D. T. Son and M. A. Stephanov, PRD 2006.

Background involving AdS plus an effective dilaton field (background). The dilaton does not back-react on the metric and plays the role of a smooth infrared cut-off.

Important feature: linear Regge trajectories for scalars.

• DIS in the soft wall model

C. A. Ballon Bayona, H. Boschi-Filho and N.B., JHEP 2008

Inspired in the 5-d soft wall model, we proposed a ten dimensional prescription for the supergravity regime:

$$\begin{aligned}\eta_\mu \langle P_X, X | \tilde{J}^\mu(q) | P, \mathcal{Q} \rangle &= (2\pi)^4 \, \delta^4(P_X - P - q) \, \eta_\mu \, \langle P + q, X | J^\mu(0) | P, \mathcal{Q} \rangle \\ &= i\mathcal{Q} \int d^{10}x \sqrt{-g} \, e^{-\varphi} \, A^m (\Phi_i \partial_m \Phi^*_X - \Phi^*_X \partial_m \Phi_i) \, .\end{aligned}$$

where g_{MN} is $AdS_5 \times W$ but coordinate z has no hard cut off. $\varphi = cz^2$

Important: fields and masses are not the same as in the hard wall

Boundary value of the gauge field represents a virtual photon with polarization η^μ

$$A_\mu(z,y)|_{z\to 0} \,=\, \eta_\mu\, e^{iq\cdot y}\,,$$

Choosing the gauge condition

$$\partial_\rho A^\rho \,+\, z e^{cz^2}\partial_z(e^{-cz^2}\frac{1}{z}A_z) \,=\, 0\,,$$

Solutions:

$$\begin{aligned} A_\mu(z,y) \,&=\, \eta_\mu\, e^{iq\cdot y}\, c\; \Gamma(1+\frac{q^2}{4c})\; z^2\, \mathcal{U}(1+\frac{q^2}{4c};2;cz^2) \\ A_z(z,y) \,&=\, \frac{i}{2}\,\eta\cdot q\, e^{iq\cdot y}\; \Gamma(1+\frac{q^2}{4c})\; z\, \mathcal{U}(1+\frac{q^2}{4c};1;cz^2)\,, \end{aligned} \tag{1}$$

where $\mathcal{U}(a;b;w) \,=\,$ conf. hypergeom. functions.

Effective maximum value for the radial coordinate

$$z_{int} \approx \frac{1}{\sqrt{c\,(1+\frac{q^2}{4c})}} \sim \frac{1}{q}\,,$$

Note: this is not an infrared cut off in space (no boundary conditions).

The 4-d center of mass energy squared $s \approx \frac{q^2}{x}$ is holographically related to 10-d energy scale $\tilde{s}$

$$\tilde{s} \le \frac{z^2}{R^2}\,s\,.$$

Since the effective interaction occurs at $z \le z_{int}$ we have

$$\tilde{s} \le \frac{z_{int}^2}{R^2}\,q^2\,(\frac{1}{x} - 1\,) < \frac{1}{\alpha'\,(4\pi g N)^{1/2}}\,\frac{1}{x}\,.$$

Supergravity approximation is valid when :

$$\tilde{s} < \frac{1}{\alpha'} \qquad \text{that corresponds to} \qquad x >> (gN)^{-1/2}$$

Normalizable solutions for the scalar field

$$\Phi_n(y,z,\Omega) = \left[\frac{2c^{\Delta-1}\Gamma(n+1)}{\Gamma(n+\Delta-1)}\right]^{1/2} \frac{1}{R^4} e^{ip\cdot y} z^{\Delta} L_n^{\Delta-2}(cz^2)\, Y(\Omega) \; .$$

with $n = p^2/4c + \Delta/2$

Initial scalar state: $n = 0$ (lowest mass in the spectrum)

$$\Phi_i(y,z,\Omega) = \left[\frac{2c^{\Delta-1}}{\Gamma(\Delta-1)}\right]^{1/2} \frac{1}{R^4} e^{iP\cdot y} z^{\Delta} \, Y(\Omega) \; .$$

Final scalar state: momentum $P_X = P + q$ so that

$$n = n_X = -\frac{P_X^2}{4c} - \frac{\Delta}{2} = \frac{s}{4c} - \frac{\Delta}{2} ,$$

$$\Phi_X(y,z,\Omega) = \left[\frac{2c^{\Delta-1}\Gamma(\frac{s}{4c} - \frac{\Delta}{2} + 1)}{\Gamma(\frac{s}{4c} + \frac{\Delta}{2} - 1)}\right]^{1/2} \frac{1}{R^4} e^{iP_X\cdot y} z^{\Delta} L_{n_X}^{\Delta-2}(cz^2)\, Y(\Omega)$$

Inserting the previous field solutions in the interaction action, calculating the matrix element of the current and plugging into the imaginary part of the amplitude:

$$\mathrm{Im} T^{\mu\nu} = 8\pi^2 \mathcal{Q}^2 \sum_X \delta(M_X^2 + (P+q)^2)[P^\mu + \frac{q^\mu}{2x}]\,[P^\nu + \frac{q^\nu}{2x}]\,(\frac{q^2}{4c})^2$$

$$\times\; (\Delta - 1)\,\Gamma(\Delta)\,[\frac{\Gamma(\frac{s}{4c} + \frac{\Delta}{2} - 1)}{\Gamma(\frac{s}{4c} - \frac{\Delta}{2} + 1)}]\;\{\frac{\Gamma(\frac{q^2}{4c} + \frac{s}{4c} - \frac{\Delta}{2})}{\Gamma(\frac{q^2}{4c} + \frac{s}{4c} + \frac{\Delta}{2})}\}^2 \,. \qquad (2)$$

The spacing between masses m_X is small compared with q. So, the sum over states X can be approximated by an integral

$$\sum_X \delta(M_X^2 + (P+q)^2) = \frac{1}{4c} \int dn\, \delta(n - \frac{s}{4c} + \frac{\Delta}{2}) = \frac{1}{4c}\,.$$

So: structure functions for scalar states in the soft wall model (in the supergravity regime):

$$F_1 = 0 \;\; ; \;\; F_2 = 8\pi^3 \frac{\mathcal{Q}^2}{x} (\Delta - 1)\, \Gamma(\Delta)\, \Big(\frac{q^2}{4c}\Big)^3 \Big[\frac{\Gamma(\frac{s}{4c} + \frac{\Delta}{2} - 1)}{\Gamma(\frac{s}{4c} - \frac{\Delta}{2} + 1)}\Big] \Big\{\frac{\Gamma(\frac{q^2}{4c} + \frac{s}{4c} - \frac{\Delta}{2})}{\Gamma(\frac{q^2}{4c} + \frac{s}{4c} + \frac{\Delta}{2})}\Big\}^2 \,.$$

Approximation: leading order contribution in c/q^2 .

$$\frac{\Gamma(\frac{s}{4c} + \frac{\Delta}{2} - 1)}{\Gamma(\frac{s}{4c} - \frac{\Delta}{2} + 1)} \approx \Big[\, \frac{q^2}{4c}\Big(\frac{1}{x} - 1\Big)\,\Big]^{\Delta - 2} \quad ; \quad \Big[\frac{\Gamma(\frac{q^2}{4c} + \frac{s}{4c} - \frac{\Delta}{2})}{\Gamma(\frac{q^2}{4c} + \frac{s}{4c} + \frac{\Delta}{2})}\Big] \approx \Big[\, \frac{q^2}{4cx}\,\Big]^{-\Delta} .$$

So

$$F_2 \approx 8\,\pi^3\, \mathcal{Q}^2 (\Delta - 1)\, \Gamma(\Delta)\, \Big(\frac{4c}{q^2}\Big)^{\Delta - 1} (1 - x)^{\Delta - 2}\, x^{\Delta + 1} \,.$$

Equivalent to the hard wall model at leading order.

Comparison of models at leading order. Non trivial compensation of factors:

Sum over intermediate states:

$$\sum_X \delta(M_X^2 + (P+q)^2\,) \;=\; \frac{1}{4c} \qquad\qquad \text{Soft wall.}$$

$$\sum_X \delta(M_X^2 + (P+q)^2\,) \;=\; \frac{1}{2\pi s^{1/2}\,\Lambda} \qquad\qquad \text{Hard wall.}$$

Amplitudes:

$$\langle P+q, X|J^\mu(0)|P,\mathcal{Q}\rangle_{HW} \;\sim\; \Lambda^{-1/2}\,s^{1/4}\,\langle P+q, X|J^\mu(0)|P,\mathcal{Q}\rangle_{SW}\,,$$

Structure functions at small x in the soft wall

$\exp(-\sqrt{gN}) << x << (gN)^{-1/2}$.

In this case there are massive string excitations, but we can approximate locally the amplitudes by those of flat space.

The four dimensional forward scattering amplitude

$$\eta_\mu \eta_\nu T^{\mu\nu} (2\pi)^4 \delta^4(q - q')$$

is identified with the ten dimensional string amplitude

$$\begin{aligned} S_{10} &= \int d^{10}x \sqrt{-g}\, e^{-cz^2} \, (\mathcal{K}\, G\,)|_{t=0} \\ &= \frac{1}{8} \int d^{10}x \sqrt{-g}\, e^{-cz^2} \{4 v^a v_a \partial_m \Phi F^{mn} F_{pn} \partial^p \Phi \\ &- (\partial^M \Phi \partial_M \Phi v^a v_a + 2 v^a \partial_a \Phi v^b \partial_b \Phi\,) F_{mn}\, F^{mn}\}\, G|_{t=0} \ , \end{aligned}$$

where v^a are the Killing vectors of the compact W space, $\mathcal{K}$ = kinematic factor, G is a flat space Veneziano amplitude

$$G = \frac{\alpha'^3 \tilde{s}^2}{64} \prod_{\tilde{\xi}=\tilde{s},\tilde{t},\tilde{u}} \frac{\Gamma(-\alpha' \tilde{\xi}/4)}{\Gamma(1 + \alpha' \tilde{\xi}/4)}$$

The 10-d Mandelstam variables $\tilde{t}$, $\tilde{s}$ are holographically related to the 4-d variables by

$$\alpha' \tilde{s} = \alpha' s \frac{z^2}{R^2} + \frac{\alpha'}{R^2} (- 3\, z \partial_z + z^2 \partial_z^2 + \nabla_W^2)$$

$$\alpha' \tilde{t} = \alpha' t \frac{z^2}{R^2} + \frac{\alpha'}{R^2} (- 3\, z \partial_z + z^2 \partial_z^2 + \nabla_W^2)$$

Note: for forward scattering $t = 0$, but $\alpha' \tilde{t}$ does not vanish.

Since $\mathcal{K}$ is real, the imaginary part of S_{10} is related to the imaginary part of G which at $t = 0$ is

$$\mathrm{Im}\, G|_{t=0} = \frac{\pi \alpha'}{4} \sum_{\ell=1}^{\infty} \delta(\ell - \frac{\alpha' \tilde{s}}{4}) \, (\ell)^{\alpha' \tilde{t}/2} \, .$$

If $\exp(-\sqrt{gN}) << x$ then we find $(\alpha' \tilde{s})^{\alpha' \tilde{t}/2} \sim 1$ then

$$\mathrm{Im}\, G|_{t=0} \approx \frac{\pi \alpha'}{4} \sum_{\ell=1}^{\infty} \delta(\ell - \frac{\alpha'\, s\, z^2}{4\, R^2}) \, .$$

Kinematic factor $\mathcal{K}$: F_{mn} associated with an incoming photon of four momentum q_μ, outgoing photon of momentum q'_μ ; Φ represents incoming and outgoing scalar states with four momentum P_μ (four dimensional plane waves).

Relevant solutions:

$$F_{0\mu}(q) = \frac{z}{2} e^{iq\cdot y} [q_\mu (q\cdot\eta) - \eta_\mu q^2] \Gamma(1+\frac{q^2}{4c}) \mathcal{U}(1+\frac{q^2}{4c}; 1; cz^2)$$
$$F_{\mu\nu}(q) = i\, cz^2 e^{iq\cdot y} [q_\mu \eta_\nu - q_\nu \eta_\mu] \Gamma(1+\frac{q^2}{4c}) \mathcal{U}(1+\frac{q^2}{4c}; 2; cz^2)\,.$$

$$\partial_\mu \Phi(-P) \partial^\nu \Phi(P) = P_\mu P^\nu \frac{2c^{\Delta-1}}{\Gamma(\Delta-1)} \frac{z^{2\Delta+2}}{R^{10}} \, |\, Y(\Omega)\,|^2 .$$

Angular normalization

$$\int d^5\Omega \, \sqrt{g_W} v^a v_a \,|\, Y(\Omega)\,|^2 = \rho\, R^2 ,$$

ρ = dimensionless quantity.

So:

$$\mathrm{Im} S_{10} \;=\; (2\pi)^4 \delta^4(q-q')\,\frac{\pi\alpha'\,\rho}{8\,R^2}\,\frac{2c^{\Delta-1}}{\Gamma(\Delta-1)}\,P_\mu P^\nu \sum_{\ell=1}^{\infty}\int dz\, e^{-cz^2}\, z^{2\Delta+3}\;\;\Gamma^2(1+\frac{q^2}{4c})$$
$$\times\;\{\frac{1}{4}\,[\,q^\mu(q\cdot\eta)\;-\;\eta^\mu q^2\,]\,[\,q_\nu(q\cdot\eta)\;-\;\eta_\nu q^2\,]\;\mathcal{U}^2(1+\frac{q^2}{4c};1;cz^2)$$
$$+\;c^2\,z^2\,[\,q^\mu\eta^\gamma\;-\;q^\gamma\eta^\mu][\,q_\nu\eta_\gamma\;-\;q_\gamma\eta_\nu]\;\mathcal{U}^2(1+\frac{q^2}{4c};2;cz^2)\,\}\,\delta(\ell\;-\;\frac{\alpha' s z^2}{4R^2})\,.$$

This leads to

$$\mathrm{Im} T^{\mu\nu}\;=\;\frac{\pi\,\rho\,c^{\Delta-1}}{8\Gamma(\Delta-1)}\;\frac{(q^2)^2}{s\,x^2}\{\;[\eta^{\mu\nu}-\frac{q^\mu q^\nu}{q^2}]\,\mathcal{A}_2+[\,P^\mu+\frac{q^\mu}{2x}\,]\;[\,P^\nu+\frac{q^\nu}{2x}\,]\,4x^2\,(\mathcal{A}_1+\frac{\mathcal{A}_2}{q^2}\,)\;\}\,,$$

where

$$\mathcal{A}_1\equiv\frac{1}{4}\Gamma^2(a)\,\sum_{\ell=1}^{\infty} e^{-cz_\ell^2}\,z_\ell^{2\Delta+2}\,\mathcal{U}^2(a;1;cz_\ell^2)\quad,\quad \mathcal{A}_2\equiv c^2\,\Gamma^2(a)\,\sum_{\ell=1}^{\infty} e^{-cz_\ell^2}\,z_\ell^{2\Delta+4}\,\mathcal{U}^2(a;2;cz_\ell^2)$$

with $a\;=\;1+\frac{q^2}{4c}$.

So, the soft wall structure functions for small x are

$$F_1 = \frac{\pi^2 \rho c^{\Delta-1}}{4\Gamma(\Delta-1)} \frac{(q^2)^2}{s\, x^2} \mathcal{A}_2$$
$$F_2 = \frac{\pi^2 \rho c^{\Delta-1}}{4\Gamma(\Delta-1)} \frac{(q^2)^2}{s\, x^2} (2\, x\, q^2) (\mathcal{A}_1 + \frac{\mathcal{A}_2}{q^2}) .$$

Leading order approximation in c/q^2:

....

we find

$$F_1 \approx \frac{\pi^2 \rho \mathcal{I}_{1,\,2\Delta+3}}{8\,(4\pi\, gN)^{1/2}\, \Gamma(\Delta-1)} \frac{1}{x^2} \left(\frac{c}{q^2}\right)^{\Delta-1}$$
$$F_2 \approx 2\, x\, F_1 \frac{\mathcal{I}_{0,\,2\Delta+3} + \mathcal{I}_{1,\,2\Delta+3}}{\mathcal{I}_{1,\,2\Delta+3}} = 2\, x \frac{2\Delta+3}{\Delta+2} F_1 \;,$$

in agreement at leading order with the hard wall structure functions.

Fermions

The soft wall model does not work as an IR cut-off for fermions.

But we can see the effect of the soft wall photon on fermions using a hybrid model, with a hard cut off $z = z_{max}$.

Initial fermionic state

$$\psi_i = \frac{\tilde{C}_i}{\Lambda^{3/2} R^{9/2}} e^{iP\cdot y} e^{\varphi/2} (\Lambda z)^{\tau+1/2} P_+ u_{i\sigma} ,$$

Final fermionic state with

$$\psi_X = \tilde{C}_X \left(\frac{\Lambda}{R^9}\right)^{1/2} s^{1/4} e^{iP_X\cdot y} e^{\varphi/2} z^{5/2} [J_{\tau-2}(s^{1/2}z) P_+ + J_{\tau-1}(s^{1/2}z) P_-] u_{X\sigma'} .$$

Fermion-photon interaction in supergravity $\lambda = \psi(z, y) \otimes \eta(\Omega)$

$$S_{int} = i \mathcal{Q} \int d^{10}x \sqrt{-g} e^{-\varphi} A_m \bar{\lambda}_X \gamma^m \lambda_i .$$

... after some steps...

$$F_2 = 2 F_1 = \pi \mathcal{Q}^2 C' \left(\frac{\Lambda^2}{q^2}\right)^{\tau-1} x^{\tau+1} (1-x)^{\tau-2} ,$$

Deep inelastic scattering when final states have higher conformal dimension: $\Delta' > \Delta$. (hard wall model)

C.A. Ballon Bayona, H. Boschi-Filho, N.B. arXiv:0712.3530 [hep-th].

In all the previous analysis the initial and final hadrons have the same dimension: $\Delta' = \Delta$.

Let us see how this affects:

current conservation $\Leftrightarrow$ transversality of scattering amplitude.

In the supergravity regime

$$(2\pi)^4\,\delta^4(P_X - P - q)\,\eta_\mu\,\langle P+q, X|J^\mu(0)|P,\mathcal{Q}\rangle \,=\, \int d^{10}x\sqrt{-g}A^m\,j_m\,,$$

where the 5-d scalar current is

$$j_m \,=\, i\,\mathcal{Q}(\Phi_i\partial_m\Phi_X^* \,-\, \Phi_X^*\partial_m\Phi_i)$$

and $A_m = (A_z, A_\mu)$ (Kaluza-Klein gauge field).

For the scalar

$$\frac{1}{\sqrt{-g}}\,\partial_m(\sqrt{-g}\,\partial^m\Phi\,)\;-\;m_5^2\Phi\;=\;0\,.$$

The 5-d mass is related to Δ of the boundary operator dual to Φ:

$$m_5^2\;=\;\frac{\Delta(\Delta-4)}{R^2}\,.$$

So, $\Delta'\;=\;\Delta$ implies equal masses and thus conservation of five dimensional current:

$$\frac{1}{\sqrt{-g}}\,\partial_m(\sqrt{-g}\,j^m\,)\;=\;0\,.$$

Note that the 4-d part of the current is not "conserved": $\partial_\mu j^\mu\;\neq 0\,.$

However, for plane wave gauge field, the 5-d supergravity interaction is equivalent to a 4-d A_μ inter. with a conserved current:

$$\int d^{10}x\sqrt{-g}A^m\,j_m\;=\int d^{10}x\sqrt{-g}A^\mu\,(j_\mu-\frac{i}{q^2}q_\mu\eta^{\nu\alpha}\partial_\nu j_\alpha)\;\equiv\int d^{10}x\sqrt{-g}A^\mu\,j_\mu^{eff}\,,$$

with $\partial_\mu j^\mu_{eff}\;=\;0$

This leads to a transversal $T^{\mu\nu}$: $\quad q_\mu T^{\mu\nu}=0=q_\nu T^{\mu\nu}$

When $\Delta' > \Delta$

$$\frac{1}{\sqrt{-g}}\,\partial_m(\sqrt{-g}\,j^m\,) = \frac{i\mathcal{Q}}{R^2}\Phi_i\Phi_X^*[\Delta'(\Delta'-4)-\Delta(\Delta-4)] \neq 0\ .$$

This would lead to non transverse $T^{\mu\nu}$ (we could consider transverse photons...)

Phenomenological approach: modified five dimensional hadronic current:

$$\tilde{j}_m \;\equiv\; j_m -\; v_m\,\frac{1}{\sqrt{-g}}\,\partial_n(\sqrt{-g}\,j^n\,)\,.$$

Possible choice of v_m that implies conserved $\tilde{j}_m$

$$v_z \;=\; 0 \quad ; \quad v_\mu \;=\; i\,\frac{(P_X-P)_\mu}{(P_X-P)^2}\frac{R^2}{z^2}\ .$$

This current leads to a transverse $T^{\mu\nu}$.

New prescription for the four dimensional current matrix element:

$$(2\pi)^4\,\delta^4(P_X-P-q)\,\eta_\mu\,\langle P+q,X|J^\mu(0)|P,\mathcal{Q}\rangle \;=\; \int d^{10}x\sqrt{-g}A^m\,\tilde{j}_m\,.$$

This interaction term reduces to an interaction with an effective four dimensional conserved current

$$\int d^{10}x\sqrt{-g}A^m\,\tilde{j}_m = \int d^{10}x\sqrt{-g}A^\mu\,j_\mu^{eff} \;=\; ... \tag{3}$$

Then we find the structure functions for a particular Δ'

$$\begin{aligned} F_1^{\Delta'}(x,q^2) \;&=\; 0 \\ F_2^{\Delta'}(x,q^2) \;&=\; \pi^2\,2^{2\Delta}|C_i|^2|C_X|^2\mathcal{Q}^2(\frac{\Lambda^2}{q^2})^{\Delta-1}x^{1-\Delta'}(1-x)^{\Delta'-2}\,[\frac{\Gamma(\frac{\Delta'+\Delta}{2})\Gamma(\frac{\Delta'+\Delta}{2}-1)}{\Gamma(\Delta'-1)}]^2 \\ &\times\;[F(\frac{\Delta'+\Delta}{2},\frac{\Delta'+\Delta}{2}-1;\Delta'-1;-\frac{1-x}{x})]^2\,, \end{aligned}$$

where $F(a,b\,;c\,;\omega)$ is the Gauss hypergeometric function.

Note that the structure functions involve the sum over all possible values of Δ'

$$F_2(x,q^2) \;=\; F_2^{\Delta'=\Delta}(x,q^2) \;+\; \sum_{\Delta'>\Delta} F_2^{\Delta'}(x,q^2)\,.$$

Conformal dimension Δ $\Leftrightarrow$ minimum number of constituents of the state.

Minimum number of constituents of a hadron is 2.

So, $\rho \equiv (\Delta' - \Delta)/2$ represents the number of extra hadrons (mesons) that could be produced by the "final state".

All terms can be related to the $\rho = 0$ ($\Delta' = \Delta$) case:

$\rho = 1$ case ("one extra meson") is simple:

$$F_2^{(\rho=1)}(x, q^2) \; = \; F_2^{(\rho=0)}(x, q^2)\, x^{-2}(1-x)^2 \, .$$

$\rho = 2$ (maximum of 2 "extra mesons"):

$$F_2^{(\rho=2)}(x, q^2) \; = \; F_2^{(\rho=0)}(x, q^2)\, \Delta^2\, (1-x)^{-2\Delta}\, \mathcal{S}^{\,2}$$

where

$$\mathcal{S} \equiv -1 + \frac{1}{x} + (\Delta+1)\ln x + \sum_{n=2}^{\Delta+1} \binom{\Delta+1}{n} \frac{(-1)^n}{(n-1)} [1 - x^{n+1}] \, ,$$

The next terms: $\rho \geq 3$ can be calculated in a similar way.

When should these corrections with $\Delta' > \Delta$ be important?

• When the center of mass energy $\sqrt{s}$ is much larger than the hadronic mass scale we expect more hadrons (more constituents) in the final state.

$s \sim q^2/x$ **So, this corresponds to small x.**

According to standard AdS/CFT dictionary we should have : one bulk supergravity field for each boundary state.

We follow a simple phenomenological approach: one supergravity field for the final state, with $\Delta' > \Delta$.
This composite state can then evolve to multi-hadronic states (not described by the model).

The supergravity approximation is valid if $x > (gN)^{-1/2}$ so for a theory with very large 't Hooft constant gN we can use this for a region where $x << 1$.

Estimate of maximum number of hadrons in final state (no relative motion, all extra hadrons with minimum mass Λ).

$$N_{max} \approx \frac{\sqrt{s}}{\Lambda} \approx \left(\frac{q^2}{x\Lambda^2}\right)^{1/2}.$$

This places the limit:

$$0 \le \rho \le (N_{max} - 1).$$

So

$$F_2(x, q^2) = F_2^{(\rho=0)}(x, q^2) + \sum_{\rho=1}^{N_{max}-1} F_2^{\rho}(x, q^2).$$

In the limit $x \to 0$ we have

$$F_2^{\rho}(x \to 0, q^2) \approx F_2^{(\rho=0)}(x \to 0, q^2) \frac{1}{x^2} \left[\frac{(\Delta)_{\rho-1}}{(\rho-1)!}\right]^2. \tag{4}$$

Then

$$\sum_{\rho=1}^{N_{max}-1} F_2^{\rho}(x \to 0, q^2) \approx F_2^{(\rho=0)}(x \to 0, q^2) \frac{1}{x^2} N_{max}^{(2\Delta-1)}. \tag{5}$$

So

$$F_2(x, q^2) \approx \pi C_0 \mathcal{Q}^2 \left(\frac{q^2}{\Lambda^2}\right)^{1/2} x^{-1/2} \, .$$

Our model is at large gN. THIS IS NOT QCD. But let us compare our results with those of hadronic physics.

For scattering at small x ($x < 0.01$) the observed total cross section depends on x and q^2 through the variable:

$$\mathcal{T} = q^2 x^\lambda / q_0^2 \, x_0^\lambda$$

where $q_0 = 1 GeV$ and $x_0 = 3 \times 10^{-4}$ and $0.3 < \lambda < 0.4$ This is called geometric scaling.

A. M. Stasto, K. J. Golec-Biernat and J. Kwiecinski, Phys. Rev. Lett. 86, 596 (2001)

E. Iancu, K. Itakura and L. McLerran, Nucl. Phys. A 708, 327 (2002)

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In terms of the structure function F_2, geometric scaling means that

$$\sigma(q^2, x) \; = \; 4\pi^2 \alpha_{EM} \, \frac{F_2(x, q^2)}{q^2}.$$

depends just on $\mathcal{T} = q^2 x^\lambda / q_0^2 \, x_0^\lambda$, with $0.3 < \lambda < 0.4$.

For $\Delta' = \Delta$ in supergravity at small x

$$\frac{F_2^{\Delta'=\Delta}(x, q^2)}{q^2} \; \sim \; (q^2 \, x^\lambda \,)^{-\Delta} \, ,$$

with $\lambda = -\frac{\Delta+1}{\Delta}$??

While the new result adding up all allowed Δ' implies at small x

$$\frac{F_2^{\Delta'=\Delta}(x, q^2)}{q^2} \sim (q^2 x^\lambda)^{-1/2} \, ,$$

with $\lambda = 1$ So the scaling gets closer to geometric scaling.

Deep Inelastic Scattering in D3-D7 scenario

Collaboration with: C. Alfonso Ballon Bayona and Henrique Boschi-Filho

Work in progress

Perspectives:

- **We learned that IR changes are apparently not enough to get closer to DIS as it is in the Physical world.**
- **How can we change our settings and look for a description of DIS as observed?**
- **Klebanov Strassler metric?**