Searching for a stringy description of QCD: experimentation with pure gauge theories

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Plan:

Preliminary comments

5-dimensional effective string

10-dimensional effective string

Concluding remarks

Settings

1. 5-dimensional Euclidean background metric

$$ds^2 = G_{nm} dX^n dX^m = R^2 \frac{h}{z^2} \left(dx^i dx^i + dz^2 \right) , \qquad h = \mathrm{e}^{\frac{1}{2} c z^2}$$

in string frame

2. Motivations

Hirn, Ruis, Satz (2005)

Karch, Katz, Son, Stephanov (2006)   Metsaev (2000)

3. Basic tool – computation of Wilson lines via Nambu-Goto action

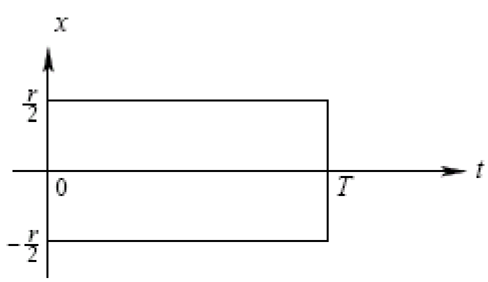
This line of thoughts is "along" the lattice theory.

Experiment 1: "Quark-Antiquark Potential "

Andreev, Zakharov (2006)

$$\langle W(\mathcal{C})\rangle \sim \mathrm{e}^{-TE(r)}.$$

Wilson loop C

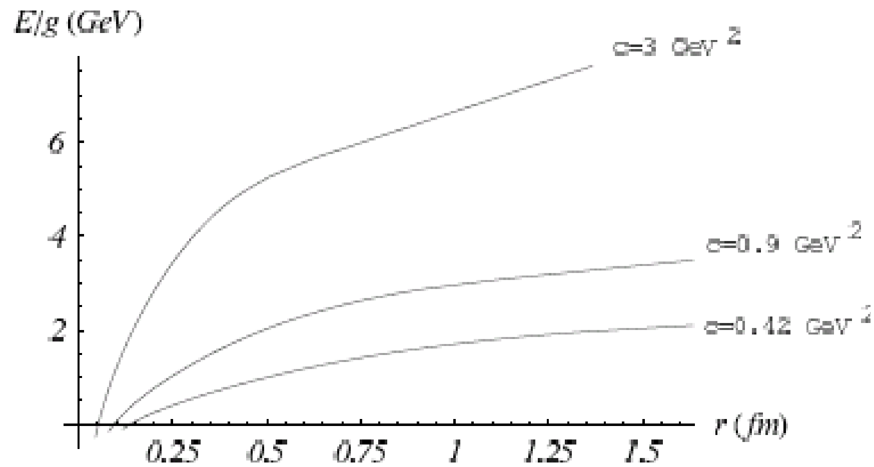


Potential

$$r = 2\sqrt{\frac{\lambda}{c}}\int_0^1 dv\, v^2 \mathrm{e}^{\frac{1}{2}\lambda(1-v^2)}\left(1 - v^4 \mathrm{e}^{\lambda(1-v^2)}\right)^{-\frac{1}{2}},$$

$$E = \frac{\mathfrak{g}}{\pi}\sqrt{\frac{c}{\lambda}}\left(-1 + \int_0^1 dv\, v^{-2}\left[\mathrm{e}^{\frac{1}{2}\lambda v^2}\left(1 - v^4 \mathrm{e}^{\lambda(1-v^2)}\right)^{-\frac{1}{2}} - 1\right]\right).$$

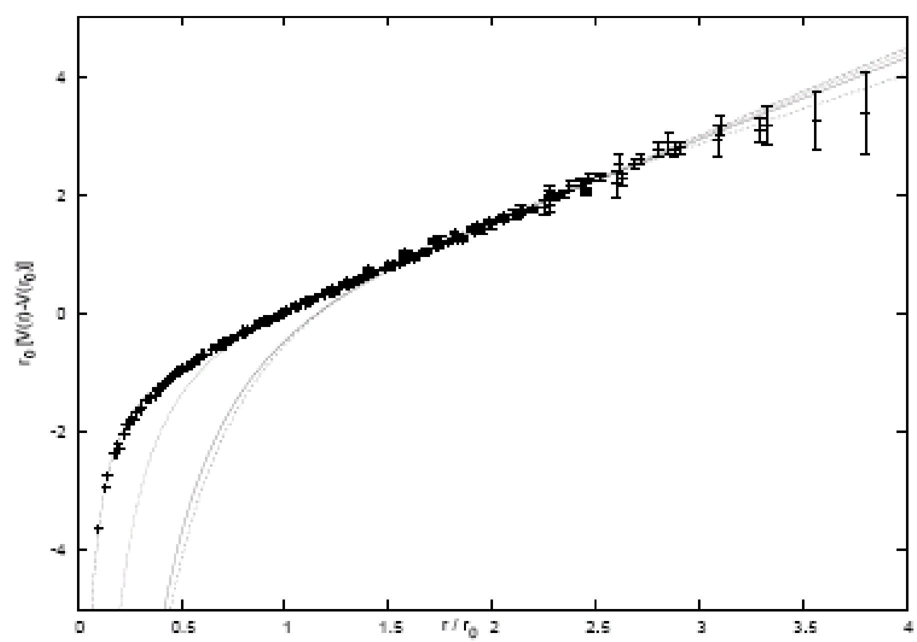
Here two parameters: *c* and $\mathfrak{g} = \frac{R^2}{\alpha'}$



c is of order 0.9 GeV^2 and g is of order 1

Matching to the lattice results

White (2007)



Experiment 2: “Gluon Condensate”

Andreev and Zakharov (2007)

$$G_2 = \left\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu} \right\rangle .$$

The goal is to compute the circular Wilson loop.
For *c=0* it was done by
Drukker, Gross, and Ooguri (1999)
&
Berenstein, Corrado, Fischler, and Maldacena (1999)

$$\ln W = -\sum_n c_n \alpha_s^n - \frac{\pi^2}{36} Z G_2 s^2 + O(s^3) ,$$

s stands for a square.

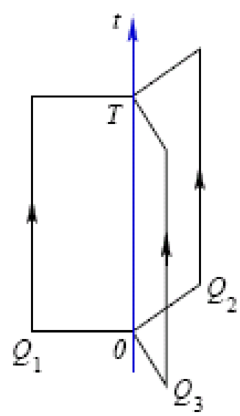
$$G_2 = 0.010 \pm 0.0023 \text{ GeV}^4 .$$

This is our main result. It is surprisingly close to the original phenomenological estimate 0.012 GeV^4 of [1], though somewhat smaller than another phenomenological estimate 0.024 GeV^4 of [17]. For comparison, the lattice calculations give bigger values like 0.04 GeV^4 [3].

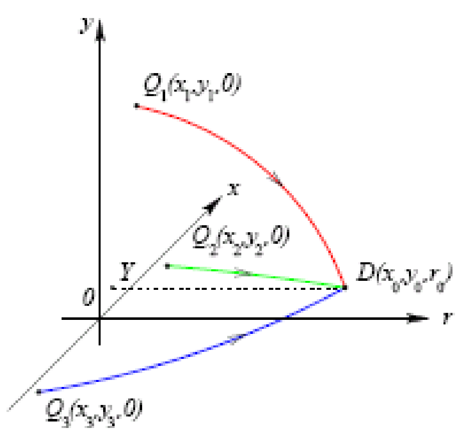
Experiment 3: "Multi-Quark Potentials"

Andreev (2008)

Baryonic Wilson Loop for SU(3)



Configuration to be considered in 5 dimensions



$$\mathcal{S} = \sum_{i=1}^{3} S_i + S_{\text{vert}} \,,$$

First, we extremize the action with respect to the location of the baryon vertex

$$\sum_{i=1}^{3} \frac{x_0 - x_i}{l_i\sqrt{1+k_i}} = 0 \,, \qquad \sum_{i=1}^{3} \frac{y_0 - y_i}{l_i\sqrt{1+k_i}} = 0 \,, \qquad \sum_{i=1}^{3} \frac{1}{\sqrt{1+k_i^{-1}}} + \frac{1}{\mathfrak{g}}\frac{\mathcal{V}'}{w}(r_0) = 0 \,.$$

Second, … with respect to the fields

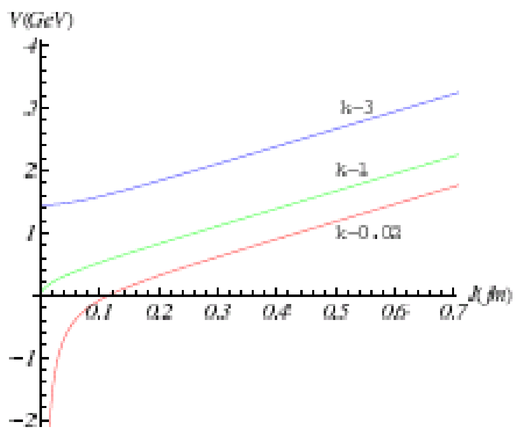
$$l_i = \sqrt{\frac{\lambda}{\mathfrak{s}(1+k_i)}} \int_0^1 dv_i \, v_i^2 \mathrm{e}^{\lambda(1-v_i^2)} \left(1 - \frac{1}{1+k_i} v_i^4 \mathrm{e}^{2\lambda(1-v_i^2)}\right)^{-\frac{1}{2}} ,$$

$$E = \mathcal{V}(\lambda) + \mathfrak{g}\sqrt{\frac{\mathfrak{s}}{\lambda}} \sum_{i=1}^{3} \int_0^1 \frac{dv_i}{v_i^2} \left[\mathrm{e}^{\lambda v_i^2} \left(1 - \frac{1}{1+k_i} v_i^4 \mathrm{e}^{2\lambda(1-v_i^2)}\right)^{-\frac{1}{2}} - 1 - v_i^2 \right] + C \,,$$

Here $k_i = \left(\frac{r_i'(0)}{l_i}\right)^2$ and $l_i = |YQ_i| = \sqrt{(x_0 - x_i)^2 + (y_0 - y_i)^2}$.

Simple Example

1. The triangle formed by the quarks is equilateral.
2. The action for the vertex is that of a point particle of mass *m* in curved space



In a generic case
at large disances

1. The Y-law

$$E = \sigma_{3q} \sum_{i=1}^{3} l_i + O(1) \,.$$

The point is that at large distances the baryon vertex contribution is of order 1. Note that this is not the case at short distances.

2. Universality of the string tension

3. Generalization to SU(N)

Experiment 4: "Spatial String Tension"

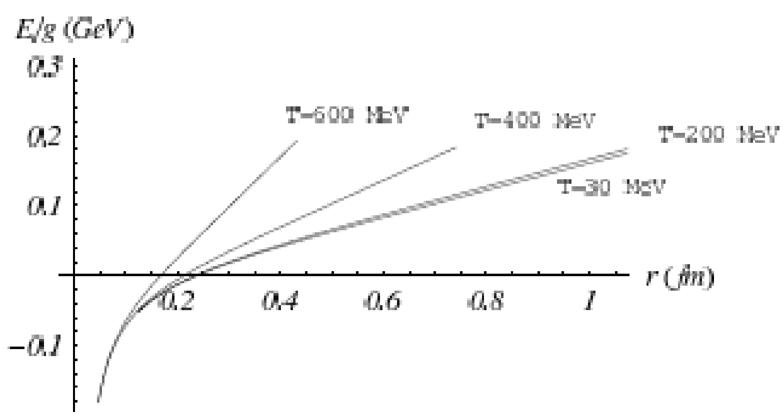
Andreev and Zakharov (2006)

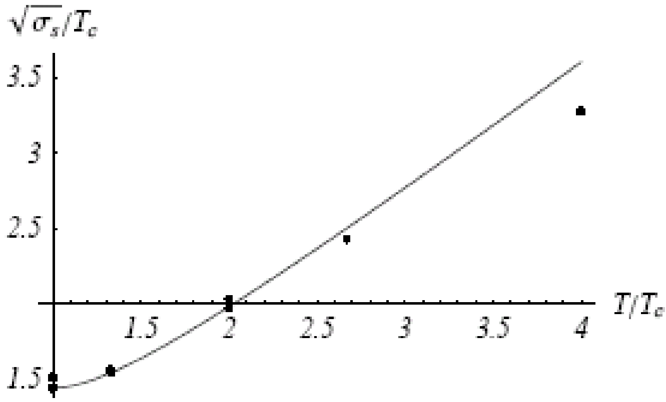
"Another" 5-dimensional metric:

$$ds^2 = R^2 \frac{h}{z^2}\left(f dt^2 + dx_i^2 + \frac{1}{f} dz^2\right), \qquad h(z) = e^{\frac{1}{2}cz^2}, \qquad f(z) = 1 - \left(\frac{z}{z_T}\right)^4,$$

where $T = \frac{1}{\pi z_T}$

The Wilson loop in question is spatial.
(it is on the xy-plane).





$$\sigma_s = \begin{cases} \sigma & \text{if} \quad T \le T_c\,, \\ \sigma\left(\frac{T}{T_c}\right)^2 \exp\left\{\left(\frac{T_c}{T}\right)^2 - 1\right\} & \text{if} \quad T \ge T_c\,. \end{cases}$$

where

$\sigma = \frac{ge}{4\pi}c$ and $T_c = \frac{1}{\pi}\sqrt{\frac{c}{2}}$.

Estimate

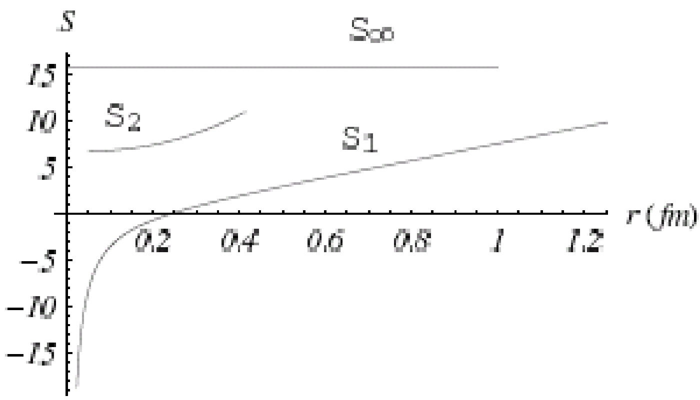
$$T_c \approx 210\,\mathrm{MeV}\,.$$

is not very impressive

Experiment 5: “Heavy Quark Free Energies, Entropies, and Polyakov Loops”

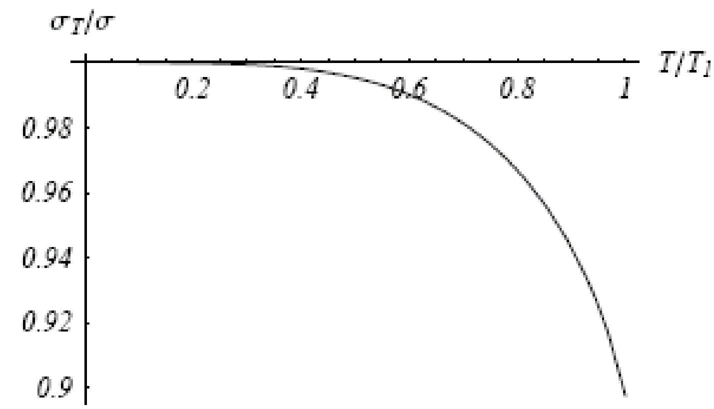
Andreev and Zakharov (2006)

Solutions to the Nambu-Goto action

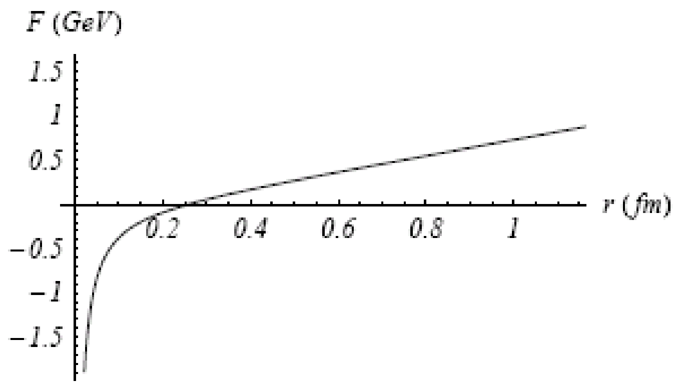


$$F(r,T) = -T \ln\langle L(\vec{x}_1) L^\dagger(\vec{x}_2)\rangle + T c(T)\,,$$

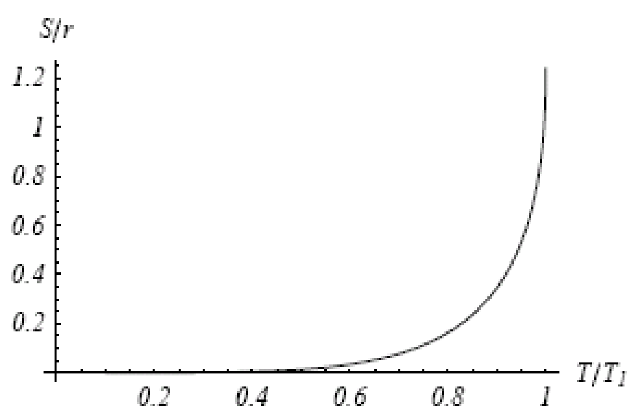
Low Temperatures



String tension

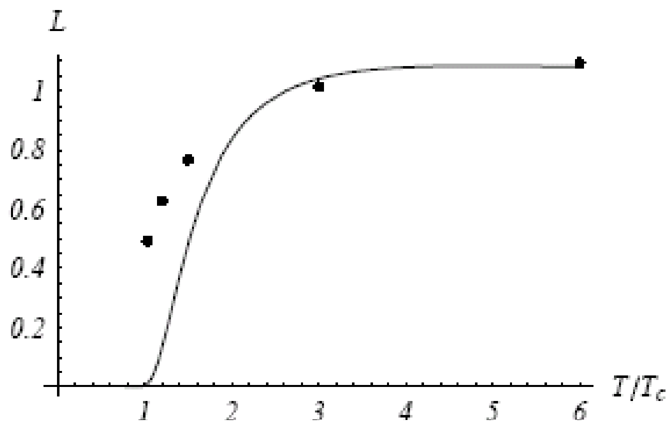


Free energy



Entropy density

Polyakov Loop (absolute value)



The normalization constant *c* is chosen as on the lattice.

Experiment 6: “Spatial String tension”

Andreev (2007)

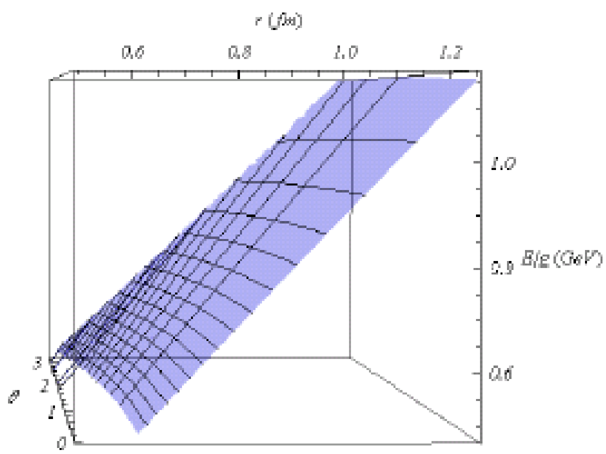
10-dimensional effective background metric

$$ds^2 = h\,\frac{R^2}{z^2}\left(f dt^2 + d\vec{x}^2 + \frac{1}{f}dz^2\right) + \frac{1}{h}\,R^2 d\Omega_5^2\,, \qquad f = 1 - \frac{z^4}{z_{\mathrm{T}}^4}\,, \qquad h = \mathrm{e}^{\frac{1}{2}cz^2}\,,$$

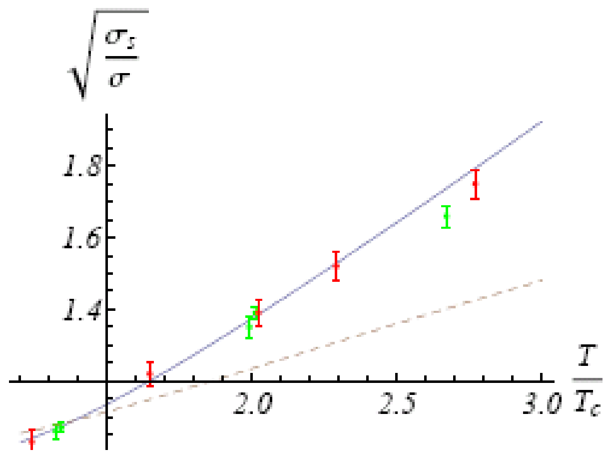
The Nambu-Goto action is

$$S = \frac{\mathfrak{g}}{2\pi} Y \int_{-\frac{r}{2}}^{\frac{r}{2}} dx\, \frac{h}{z^2}\sqrt{1 + \frac{1}{f}\,z'^2 + \frac{z^2}{h^2}\,\Theta'^2}\,,$$

The potential as a function of two variables



For SU(2), SU(3) and a hadronic gas model



Conclusions:

1. The ratio is independent of N.

2. At large distances the pseudo-potential is, to leading order, described via 5-dimensional theory.

At short distances, the Coulomb term is dependent of angles in internal space. Maldacena (1998).

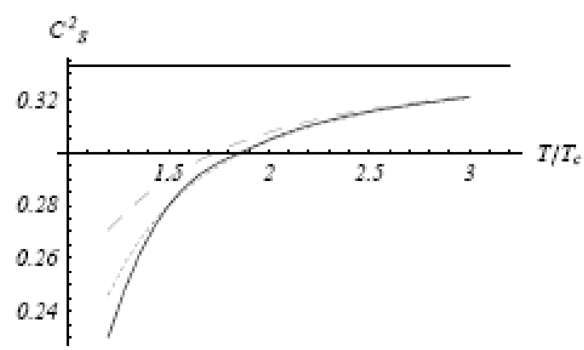
Experiment 7: “Thermodynamics”

Andreev (2007)

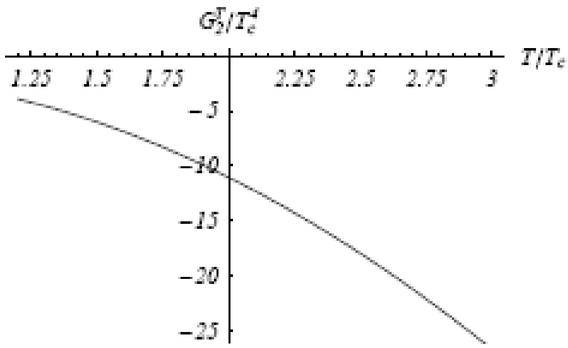
The entropy density

$$s(T) = s_0\, T^3 \exp\left\{ -\frac{1}{2}\frac{T_c^2}{T^2} \right\},$$

The speed of sound



The gluon condensate $G_2(T) = G_2 + 4p - T\,s\,,$



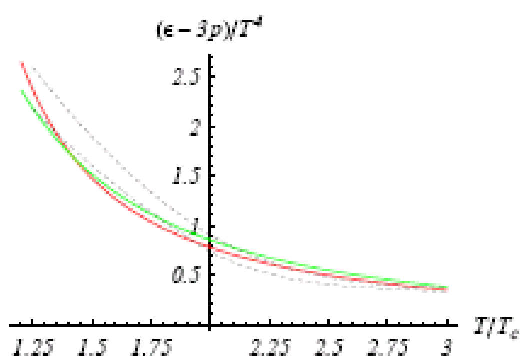
$$s_0 = \frac{32}{45}\,\pi^2 \approx 7.0$$

From the leading term in *s*

$$s_0 = 6.8 \pm 0.3\,.$$

From the lattice data

The interaction measure



Conclusions:

1. Effective string theory description vs supercomputers?

It seems YES but in a very special temperature window
$1.2\ T_c < T < 3\ T_c$.
(N is arbitrary and finite)

2. From mathematical point of view it looks like a “baby version” of …

Can we trust the computations?

At least, the string tensions should be OK.
The quasi-classical analysis results in the Luscher term which is subdominant at large distances.

3. No quarks. So it is still FAR from the real world.